"ANCIENT WORLD"

STEM TEACHING

LESSON PLANS

How Ancient World can teach us about STEM Education

Four Lesson Plans inspired from Ancient Times.

"SAVING NICOSIA FROM THE ENEMIES"

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"A JOURNEY TO THE CENTRE OF THE EARTH"

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"WHAT IS BEAUTIFUL"

-

"DO NOT GET DRUNK"

“SAVING NICOSIA FROM THE ENEMIES”

The iconic fortification walls of Nicosia were built by the Venetians in 1567. The walls have the shape of a regular hendecagon with a total circumference of approximately 5 kilometers. The Venetian walls were a huge building for the time and conditions of Cyprus, which, however, leaves many questions as to how it was constructed. Unlike a regular hexagon, octagon, decagon or even dodecagon, the regular hendecagon does not belong to the shapes that can be constructed with a ruler and compass. The length of its side is almost never a whole number, nor can its angles be calculated exactly, especially when we consider the means that the engineers and architects of that time had.

How did the architect of the walls, Giulio Savorniano, manage to plan, measure on the ground, and build with his team such a perfect, symmetrical, and large building, without the use of satellite images, laser tools and drones, so he could see the result from above? What methods did they use to approach this construction and achieve such a beautiful and perfect circular effect?

After studying the basic properties of the hexagon, students will focus on the approximate methods that -perhaps- the engineers and architects of that time considered.

In this activity students will be introduced to the 2-dimentional shapes, polygons, regular polygons, and their properties. To do so, they must learn to use the basic geometrical instruments efficiently and accurately but also use other means to construct larger 2-dimentional shapes on the ground. They will get to learn which regular polygons are constructible by geometric instruments and which are not. What approximation methods exist, especially for the regular hendecagon?

Figure 1: Plan of the walls of Nicosia. We distinguish the perfect circular shape with the inscribed regular hendecagon.

<u>Expected Outcome:</u>
The physical construction of a regular hendecagon, both on paper (in class) as well as (on a large scale) on the floor of the school’s yard.

Questions to be investigated:

1. *What are regular polygons?*
2. *Properties of regular hendecagon (eleven-sided polygon).*
3. *How did the ancient Venetian builders work back in 1567 to be able to construct such a large-scale regular hendecagon (of 5km radius) without the use of lasers and satellites?*
4. *Existing approximation methods for the construction of a regular hendecagon.*

Guidelines for Teachers:

1. *What are regular polygons?*

There are several published articles and reports on our chosen topic, as well as mathematical models that explain the properties and the constructional path followed by various approximation methods for constructing the regular hendecagon.
From our geometry lessons, we know that the regular hendecagon (from the Greek words eleven and angle) is a polygonal shape in the plane with eleven equal sides and eleven equal angles (vertices). The interior angles of any hexagon add up to exactly 1620 degrees. Since all its sides and all its angles are equal to each other, each interior angle is equal to 1620 degrees / 11 and approximately equal to 147 degrees 16’ 22’’. Its central angle is equal to 360 degrees / 11 or 32.7272… degrees. (Figure 2).

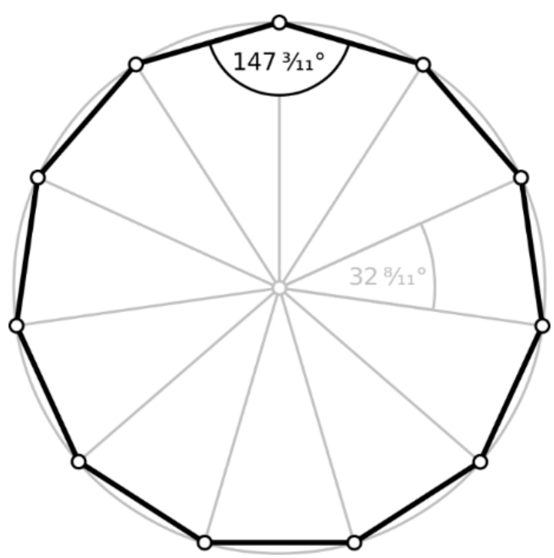


Figure 2: Regular hexagon, corner of regular hexagon and center corner

It is a special shape that is not often found in constructions in our daily life. We found that due to its complexity it has been used by various countries as a currency. A Canadian dollar (Figure 3), an old 1981 US dollar (Figure 4), a Madagascar 50- Ariary coin (Figure 5) and an old Indian 30- pence coin (Figure 6) has a regular hexagon shape.

Figure 3: Canadian Dollar

Figure 4: Old American Dollar

Figure 5: Madagascar Coin

Figure 6: Indian coin with value of two rupees

Because number eleven (11) is not a Pierpont prime (it is not of the form 2v.3v+1, where v=non r=negative integer, e.g.2,3,5,7,13,17,19… etc.) the regular hendecagon cannot be constructed even with a calibrated ruler and caliper or the method of Trichotomising the corner (construction method from the ancient Greeks, "nefsis").

2. *Properties of regular hendecagon (eleven-sided polygon).*

Definition of a Hendecagon
A hendecagon is a polygon with eleven sides and eleven angles. The term "hendecagon" originates from the Greek words "hendeka," meaning "eleven," and "gonia," meaning "angle." It is a regular polygon, meaning that all its sides and angles are equal.

Properties of a Hendecagon

- Side Length: In a regular hendecagon, all sides have the same length, denoted as "s."
- Interior Angles: The sum of the interior angles in any hendecagon is given by the formula (11 – 2) × 180 degrees = 1980 degrees. Therefore, each interior angle measures approximately 162.86 degrees.

- Exterior Angles: The exterior angles of a hendecagon add up to 360 degrees, as is the case with any polygon.
- Symmetry: A hendecagon has 11 lines of symmetry, each passing through its center and connecting opposite vertices.
- Diagonals: The number of diagonals in a hendecagon can be calculated using the formula n × (n – 3) / 2, where "n" represents the number of sides. Thus, a hendecagon has 55 diagonals.
- Area: The formula for calculating the area of a regular hendecagon is A = (11 × s²) / (4 × tan(?/11)), where "A" represents the area and "?" is a mathematical constant (approximately 3.14159).

3. *How did the ancient Venetian builders work back in 1567 to be able to construct such a large-scale regular hendecagon (of 5km radius) without the use of lasers and satellites?*

   An approximate method (Image 11), is described in detail by T.Drummond in 1800 AD. Following the engraving by Anton Ernst Burkhard and Birckensteinin 1698. Is a simple method that forms the regular hendecagon with a greater deviation.

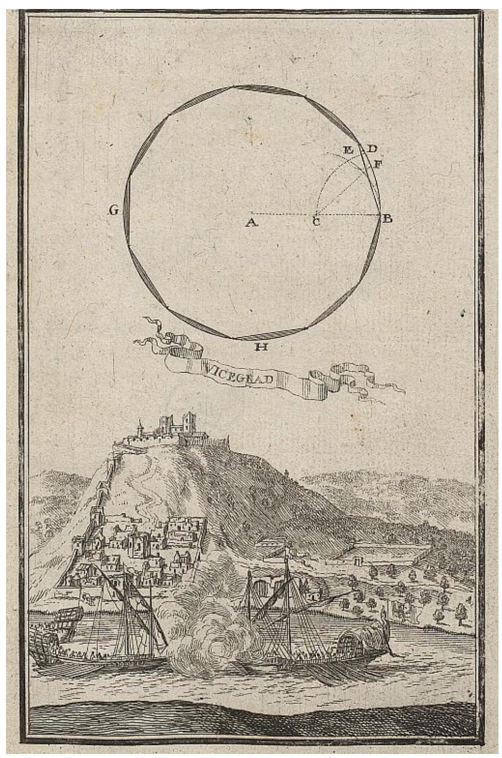


Figure 7: Etching by Anton Ernst Burkhard and Birckenstein (1698)

By successively constructing diagonals and equal arcs, students will end up with a fairly good approximation of the side of a regular hendecagon. Then, with consecutive equal arcs on the circumference of the initial circle, the final shape will be formed.

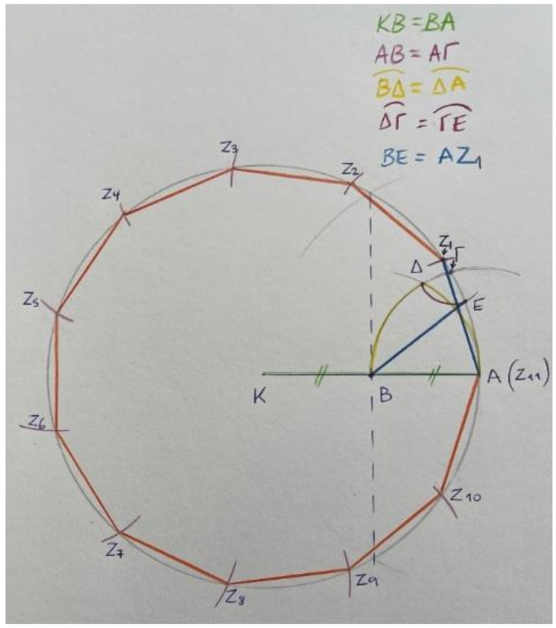


Figure 8: The approximate method, with a ruler and calliper on paper.

4. *Existing approximation methods for the construction of a regular hendecagon.*

Our first case is both the most obvious and mathematically simplest: calculating on paper the dimensions of a regular hendecagon by using simple calculation methods from geometry and trigonometry. Knowing that the length of the circumference of the circle is given by the formula Γ=2πR and therefore for Γ=5000 meters we calculate the radius of the circumscribed circle at 795 meters.

We can also easily calculate the central angle of this circle by dividing 360 degrees by 11, which equals 32.7273 degrees (rounded to four decimal places). Since the triangle KAB is isosceles (KA=KB=R) using the sine of half the central angle formula, we find the length of the line segment MB, and then the length of the side of our regular hendecagon, equal to 445 meters. The architects and engineers, quite simply, moving clockwise at an angle of 147 degrees and a distance of 445 meters each time, managed to construct the regular hendecagon with quite a high degree of accuracy.

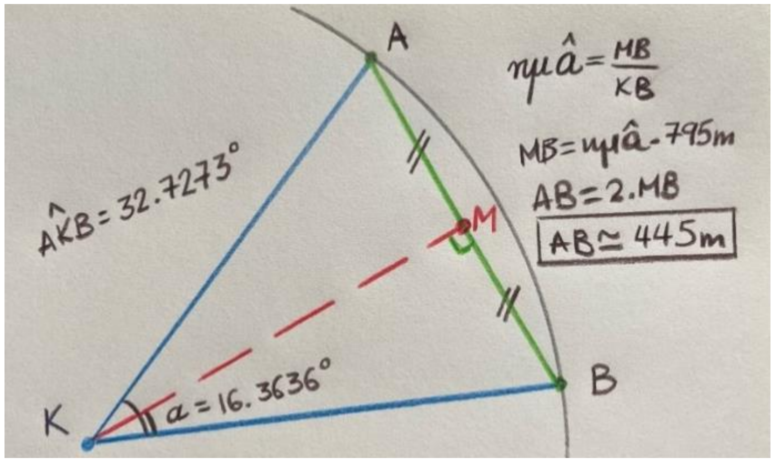


Figure 9: Calculating the side of a regular hexagon (at actual distance).

We have also found two alternative construction methods, quite more complex, but which give very good approximations. The first approximation method (which we implemented as a construction in the central yard of our school) includes construction of a circle with radius OA, mid-perpendicular of the straight segment ΟΓ (at point Δ), mid-perpendicular of ΕΔ at point Z and point H the trace of the straight-line OZ at the circumference of the circle.

With point B as centre and radius BH we find the point of intersection Θ with the circle. With the point Θ as the centre and the length ΘH now a fixed radius, moving successively and iteratively we finally form the normal hexagon with relatively very small deviations (7 sides with a deviation of 0.37% and 4 sides with a deviation of 0.61%).

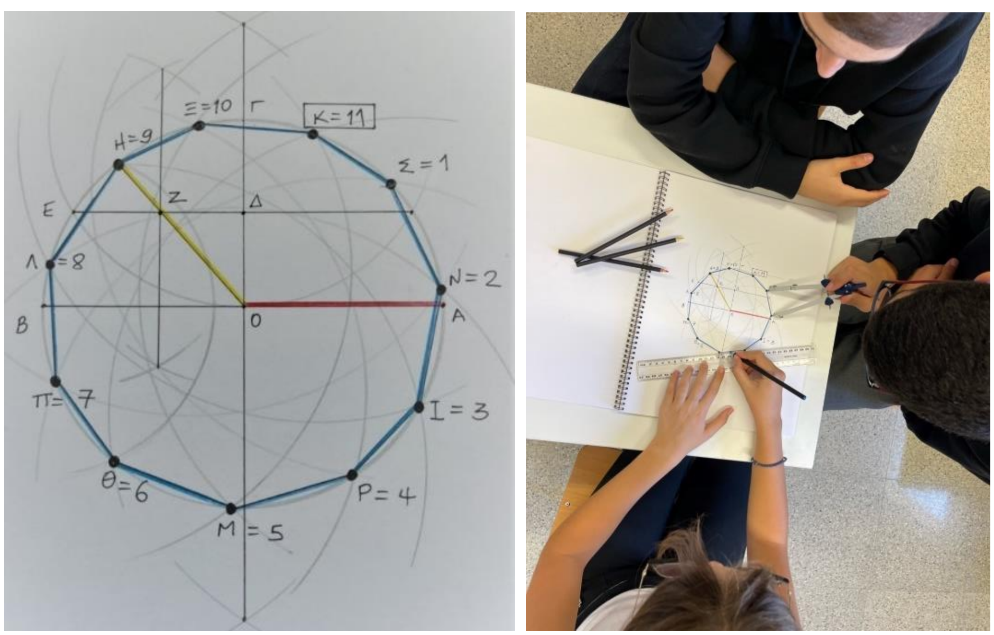


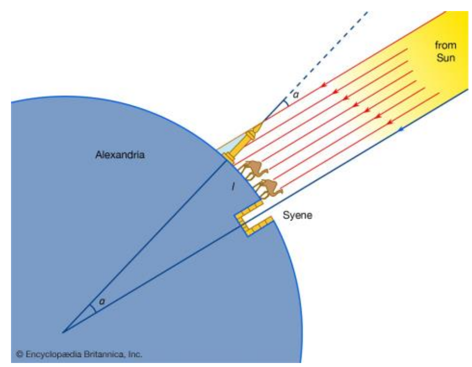
Figure 10: The course of construction of the first approximate method, with ruler and calliper on paper

Although considerably more complicated, the first approximation method is the one that will trigger students' interest, due to its high accuracy in the final result. Using simple materials, such as rope, calibrated ruler and coloured chalks, they will be able to construct a regular hendecagon (inscribed in a circle of radius of 5 meters) along with all the auxiliary lines and circular arcs with each side of the hendecagon approaching the ideal 2.82 meters.

By the students: **Kyprianos Iacovou, Avraam Olymbiou and Liudmila Bakhtiiarian** (students of B02 Class, Year 2, of Pagkyprion Gymnasion in Nicosia, facilitated and coordinated by their Mathematics Teacher (Department of STEAM Education), **Mr. Yiannis Lazarou.**

"A JOURNEY TO THE CENTER OF THE EARTH"

More than 2,000 years ago Eratosthenes compared the position of the Sun's rays in two locations and managed to calculate the Earth's circumference with reasonable accuracy. In this activity students will conduct a simpler experiment that is based on Eratosthenes's experiment. In the process they will study the curvature of the Earth, conclude as to its spherical shape and measure its radius. During this investigation they will study several other questions related to this.



Expected outcome:

Earth is a spheroid with a radius of approximately 3600 km.

Eratosthenes's experiment

Eratosthenes became aware of a well situated in the Egyptian city of Swenet (at a latitude of 23.5 degrees north), known as Syene (in Aswan), on the Nile River. During the summer solstice, which occurs between June 20 and June 22, the Sun's rays would shine vertically into the deep pit at noon. This unique phenomenon would solely illuminate the water at the bottom, while the sides of the well remained unaffected, thus providing evidence that the Sun was positioned directly overhead. It is worth noting that Syene is located in proximity to the Tropic of Cancer, specifically, which represents the northernmost point where the Sun is ever directly overhead at noon.

At the same time, in Alexandria, an erected a pole was observed to cast a shadow. This difference in shadows, demonstrated that the Sun was not directly above but rather slightly to the south. By acknowledging the Earth's curvature and possessing knowledge of the distance between the two cities, Eratosthenes was able to compute the circumference of the planet.
For more information about Eratosthenes experiment, students can view this video.
https://youtu.be/f-ppBtuc_wQ?si=NJcLjJugmPyNxUAH

Questions to be investigated:

1. *Relative Earth-Sun motion.*
2. *Similar Shapes*
3. *Sun rays' angle of incidence, in different parts of the globe.*

Guidelines for Teachers:

1. *Relative Earth-Sun motion.*

   Using a torch, a globe and a thread, students may construct a model to represent the sun rays and the relative Sun-Earth motion, as the pictures indicates. Earth rotates around its axis and therefore, day and night is created. It also orbits around the Sun, while Sun is stationery.

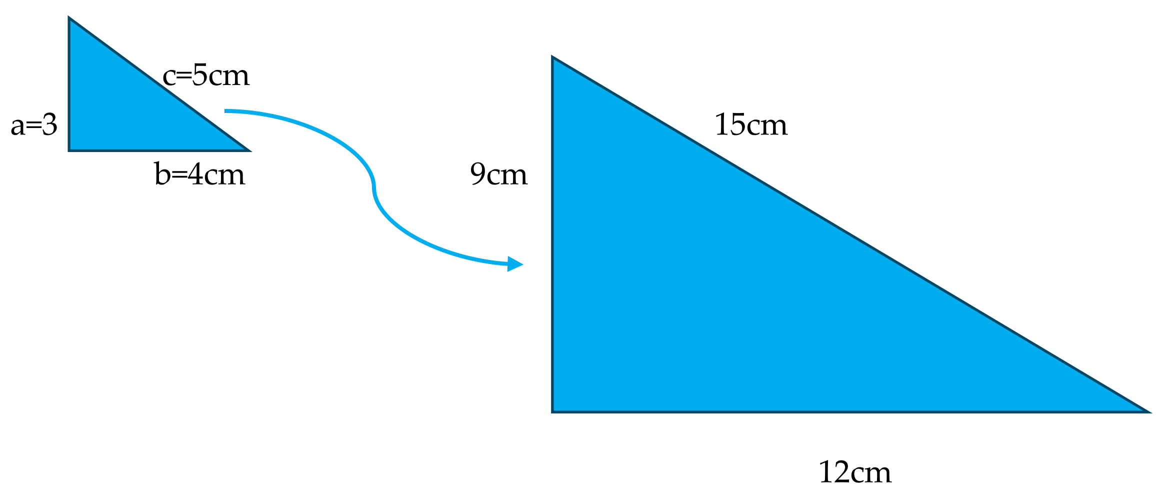
   (See **Capturing the Sunlight**)

2. *Similar Shapes*

   Students need to study the mathematical concept behind similarity. Two shapes are similar when one is an enlargement of the other. This means that if you multiply one side with a constant value, then all other sides are multiplied by the same constant.

   For example,

   "A triangle with dimensions a=3cm, b=4cm and c=5cm is enlarged three times. This means that sides a, b and c will correspond to new sides that measure 9cm, 12cm and 15cm".



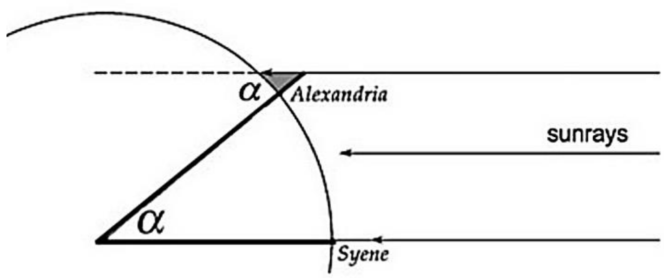
The enlargement of one shape into the other is called ratio. The usual way to calculate the ratio is to divide one side of the enlarged shape with the corresponding side of the original triangle.

3. *Sun rays' angle of incidence, in different parts of the globe.*
   Using a Globe and a torch, students can easily notice that in some areas the sun rays are vertically oriented towards the Globe and at the same time in other places, there is a different angle of incidence that creates a shadow.
   (See **Capturing the Sunlight**)

Main Investigation:

Firstly, students need to pick a sunny day, close to the solstice day. They will place a pillar as vertically as possible on a flat straight surface and they will notice that the pillar creates a shadow whose length changes as the Sun moves during the day. They will also notice that the shortest length of the pillar is marked during the noon.

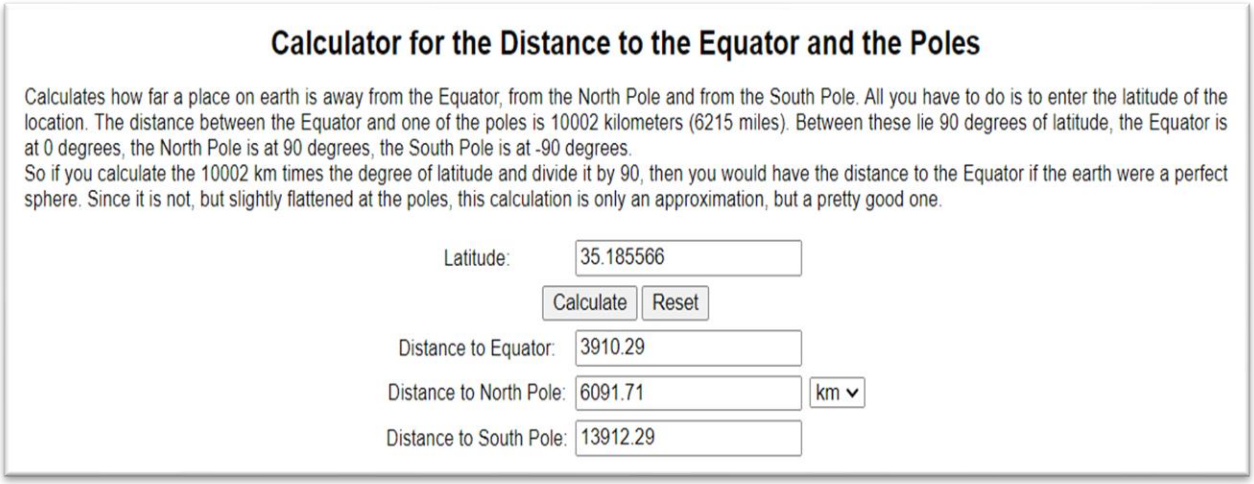
According to their previous investigation, at noon, exactly Southern (or Northern) on the equator no shadow is detected. They are now creating a similar experiment as to the one that Eratosthenes contacted. The pillar that the students placed is what Alexandria was for Eratosthenes and Syene is now the distance of the pillar from the equator.



Distance from the Equator

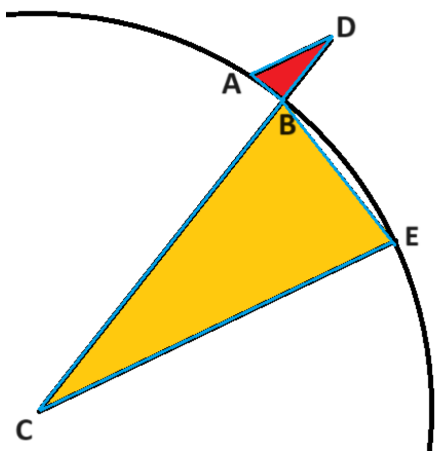
The only extra information they will need is the distance of the pillar from the equator. Eratosthenes measured the distance from Syene to Alexandria by hiring "walkers", who measured a distance by walking. In the students experiment the exact distance of the pillar from the Equator can be calculated using the website: https://rechneronline.de/earth-radius/distance-equator-pole.php

By entering their towns latitude, the distance from the Equator is automatically calculated.



<u>Similar Triangles</u>

The triangle that is created from the Shadow and the Pillar, is similar to the one that is created from the pillar's projection towards the Earth's Centre and the point E. Therefore, the triangle ABD is similar to EBC, because AD//CE.



AB: Pillar's Shadow

BD: Pillar

E: Equator Point

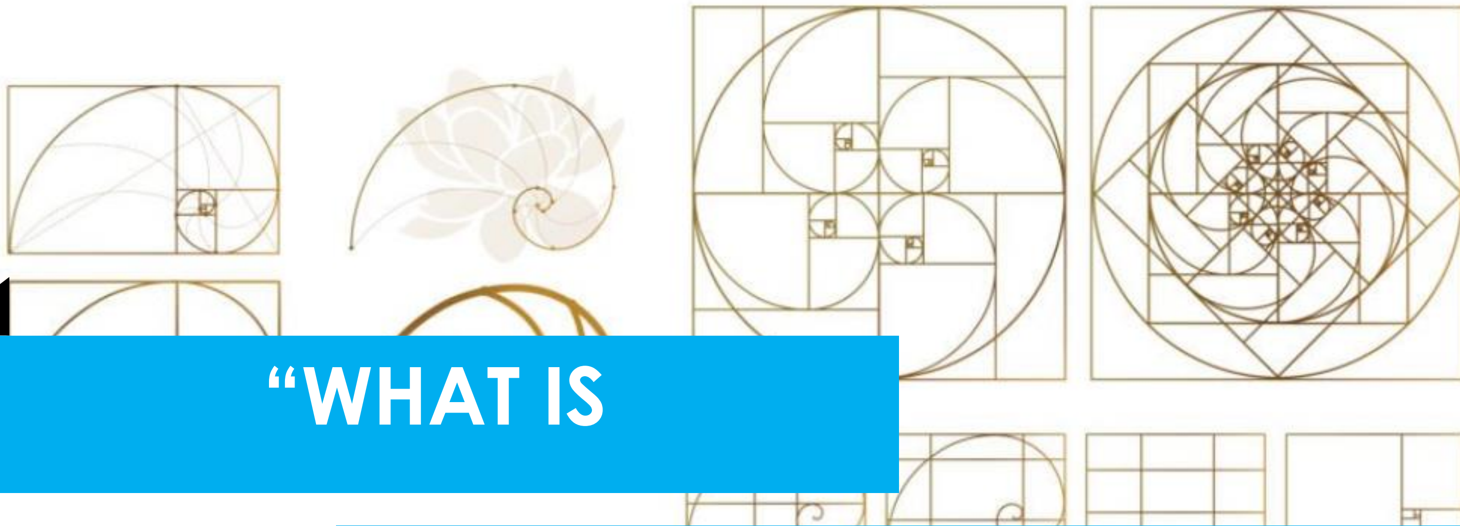
CB: Earth's Radius

Students can now estimate the enlargement needed for ABD triangle to be equal to BCE. To do so the need to divide BE over AB. This is the ratio needed to be multiplied with the pillar's height to be equal to BC, i.e., the Earths Radius
Students can use the following table to fill all necessary details and find the Earths radius.

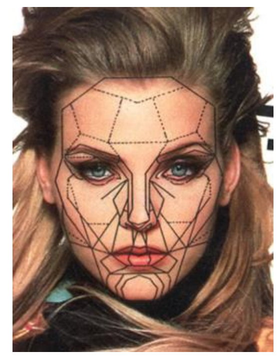
Shadow Length (AB)	Time		Distance from Equator (BE)	Pillar length (BD)	Ratio $\frac{BE}{AB}$	Ratio x Pillar $BD \cdot \frac{BE}{AB}$
						Earth's Radius

Conclusion

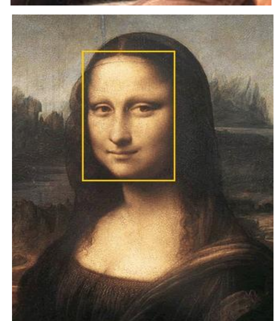
Through this activity students calculated the Earth's radius, an activity based on Eratosthenes experiment, who managed to measure the Earth's circumference 2000 years ago. Through this simplified activity they studied about light, Earth's and Sun relative movement and similar triangles.



"WHAT IS BEAUTIFUL"



In this activity students will investigate the mathematics hidden behind the Golden Ratio, also known as the Divine Proportion. They will research its origins observed in everything from plants, snail shells, the Galaxy to architecture application and even body aesthetics. Given its pervasive presence in various aspects of society, some experts suggest that humans may subconsciously assess one another based on this standard. This prompts the question: "How is the Golden Ratio connected to what is beautiful?"



<u>Expected outcome:</u>
When a line is divided into two segments in a ratio of 1:1.618, it establishes the ideal proportion. This generates the Golden Rectangle, the Fibonacci sequence, and the spiral shape.

<u>Application of Golden Ratio</u>

From the striking profile of Nefertiti to the majestic Parthenon in Rome, historical recognition of beauty has often been linked to harmonious proportionality. Taking this concept, a step further, some individuals propose that the ancient mathematical principle known as the Golden Ratio can serve as a metric for evaluating a person's attractiveness. In this month's post, we delve into a comprehensive exploration of the Golden Ratio and examine whether it should be considered a benchmark for facial beauty.

Questions to be investigated:

1. What do shapes recognised as beautiful, have in common?
2. How is Fibonacci sequence connected to Golden Ratio?
3. Connection between Fibonacci sequence and Spiral Shape?
4. What applications of Golden Ratio and Spiral are found in Nature, in architecture and in Human Body?

Guidelines for Teachers:

- *What is Golden Ratio.*

The golden ratio, also known as the golden number, golden proportion, or the divine proportion, is a ratio between two numbers that equals approximately 1.618. Usually written as the Greek letter phi "Φ", to honor Phidias, one of the architects of Parthenon, who used Golden Ratio in building Parthenon.

- *Why is it considered to be an ideal beauty ratio?*

Proportion and Harmony:
The golden ratio is believed to represent a proportion that is aesthetically pleasing and harmonious. When certain facial or body features adhere to the golden ratio, they are thought to create a sense of balance and symmetry, which many people find visually appealing.
Historical and Cultural Significance:
Throughout history, artists and architects have incorporated the golden ratio into their work. Some believe that this historical prevalence contributes to the perception of the golden ratio as an ideal standard of beauty. For example, it is said to have been used in the design of famous artworks, such as the Parthenon in Athens.
Nature's Patterns:
The golden ratio is found in various patterns in nature, such as the arrangement of leaves, petals, and spirals in shells. Some argue that because these patterns are found in aesthetically pleasing natural forms, applying the golden ratio to human features may enhance perceived beauty.

➢ *How is Fibonacci connected to Golden Ratio?*

The Fibonacci sequence is a series of numbers in which each number is the sum of the two preceding ones, usually starting with 0 and 1. So, the sequence begins 0, 1, 1, 2, 3, 5, 8, 13, 21, and so on.

The connection between the Fibonacci sequence and the golden ratio lies in the ratios of consecutive Fibonacci numbers. As you progress through the Fibonacci sequence, the ratio of consecutive numbers begins to converge towards the golden ratio. More precisely:

$$\lim_{n \to \infty} \frac{F_{n+1}}{F_n} = \Phi$$

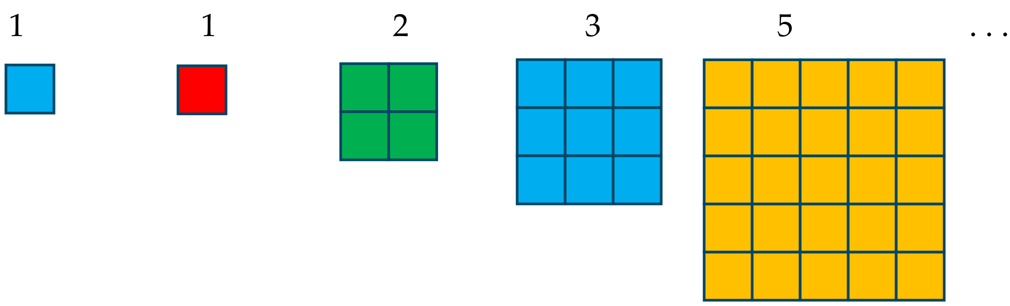
Here, represents the nth term in the Fibonacci sequence.

In simpler terms, as you take larger and larger Fibonacci numbers and divide them, the result approaches the value of the golden ratio.

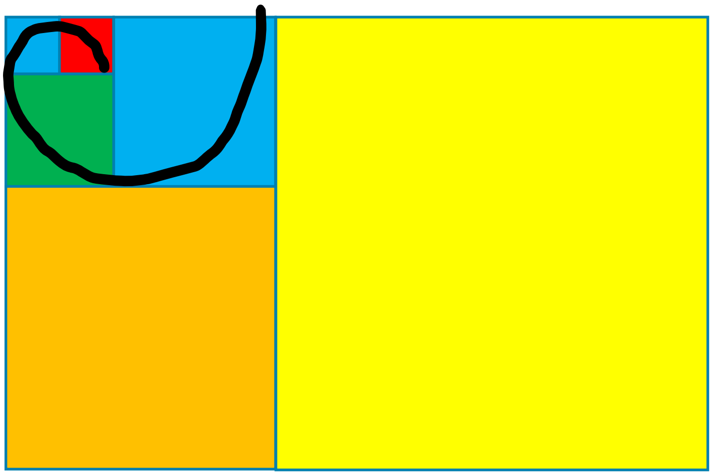
This connection between the Fibonacci sequence and the golden ratio can be observed in various aspects of nature, art, and architecture. For example, the arrangement of leaves on a stem, the spirals in a pinecone or sunflower, and the branching of trees often follow patterns related to the Fibonacci sequence and the golden ratio. Additionally, some artists and architects have incorporated the golden ratio into their work, believing it provides a sense of aesthetic harmony and balance.

➢ *How is Fibonacci connected to spiral shape?*

The connection between Fibonacci numbers and spiral shapes is a result of the self-replicating nature of the Fibonacci sequence and the influence of the golden ratio. The spiral growth pattern allows for efficient packing of structures, and it appears in various forms in nature, showcasing the mathematical beauty inherent in the natural world. The most common representation of Fibonacci Sequence into spiral shape is through the construction of squares, as the Fibonacci numbers indicate. For example,

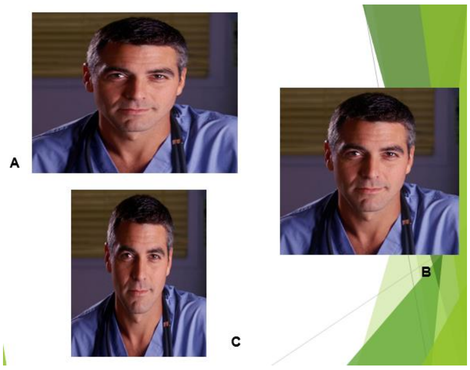


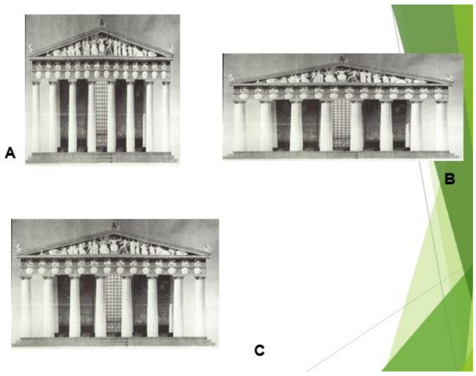
By joining the squares as the following figure indicates, and applying an inner tangent, the spiral is then generated.

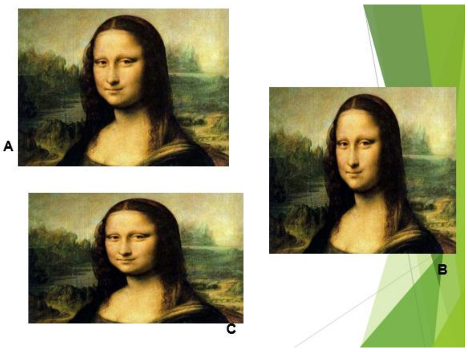


- Main investigation:
    - The Golden Rectangle

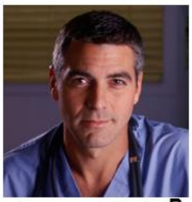
Students will be asked to choose which of the following pictures is considered to be more harmonic, or more beautiful. The pictures on each group are the same, but their proportions vary, so as only one of them has the golden hidden in its shape. (Presentation 1)







It is expected that the vast majority of students will choose respectively:


B


C

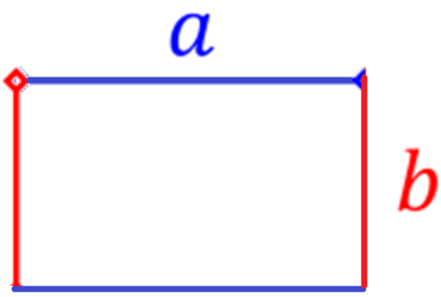

A

The teacher will boost students' curiosity by asking them "Why do we all agree as to what is beautiful"? One would expect that beauty should be subjective, not objective. And if what our mind considers as beautiful, is objective, then is there a way to count this objectiveness?

Main Activity:

Students will be asked to draw a rectangle on a paper, so to have the ideal proportions and be the most beautiful rectangle they ever drawn. Afterwards a discussion will take place, as to the fact that the rectangles have different sizes, but they all seem to have the same proportions. Therefore, the students will divide the bigger side over the smaller one and share their answer with the class. All answers are expected to be close to 1.6 .

$$\frac{\boldsymbol{a}}{\boldsymbol{b}} = \frac{16}{10} = \frac{8}{5} = \cdots \frac{\mathbf{1.6}}{\mathbf{1}}$$



The rectangle with dimensions having ratio of 1.6 is called Golden Rectangle and is found in many paintings, sculptures, buildings, in nature and even in human body.

Students are now asked to search the website for Golden Rectangle applications and make a short presentation of their findings.

Students are expected to find the Golden Rectangles in a variety of artworks such as Mona Lisa, Greek Ancient Sculptures, the Parthenon, Taj Mahal, the Great Pyramid of Giza and many other. Students now need to understand that not all artists knew about the Golden Ratio.

They created art, as to what they thought was beautiful and afterwards people found out that their work includes the Golder Rectangle in its design.

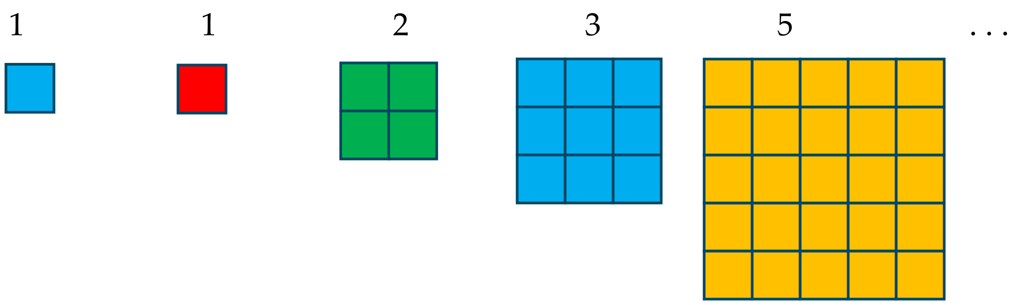
- The Spiral

Students are now asked to determine the next two numbers on the Fibonacci Sequence

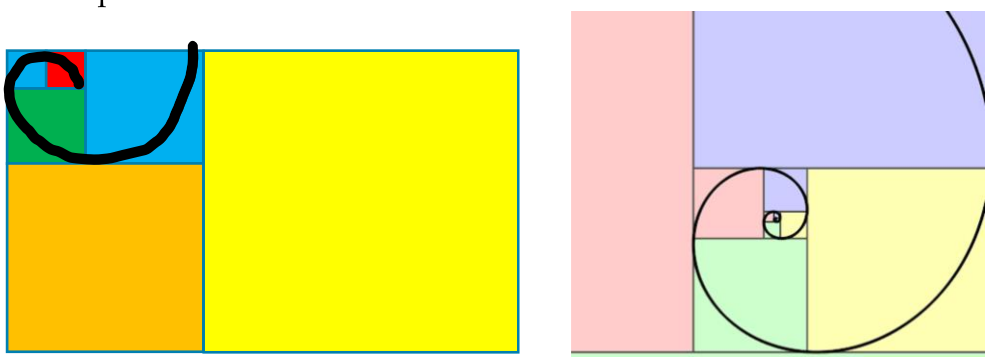
1, 1, 2, 3, 5, 8, 13, 21, 34, _____, _____

The students now understand that the logic behind this sequence is that you add two continuous numbers to find the next one. Afterwards the students are asked to discover where the Golden Ration is hidden in the Fibonacci sequence. The students are expected to find out that if one divides one term with the previous one, the ratio is a number close to 1.6 i.e., the Golden Ratio.

The students now can construct on a piece of paper the Fibonacci sequence in rectangles and arrange them in this order:



By joining the squares as the following figure indicates, and applying an inner tangent, the spiral is then generated. They are ready now to watch the slideshow that reveals how the Fibonacci Sequence is related to the spiral.



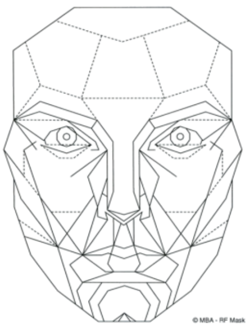
The students have all knowledge needed to search for the spiral in nature and present their work in class. A variety of findings are expected to arise, most of which are found in the following figure.

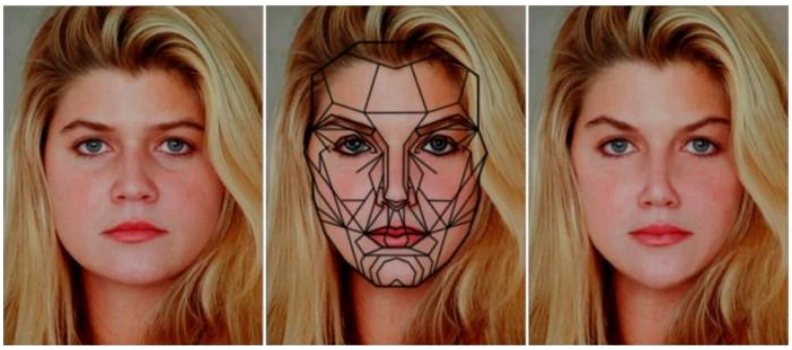
- The Human Body

The golden ratio is reflected in various aspects of the human body, including facial features, body proportions, hand and finger lengths, and skeletal structures. Students can measure different parts of their body in order to reveal the Golden Ratio. For example:

Facial Features:

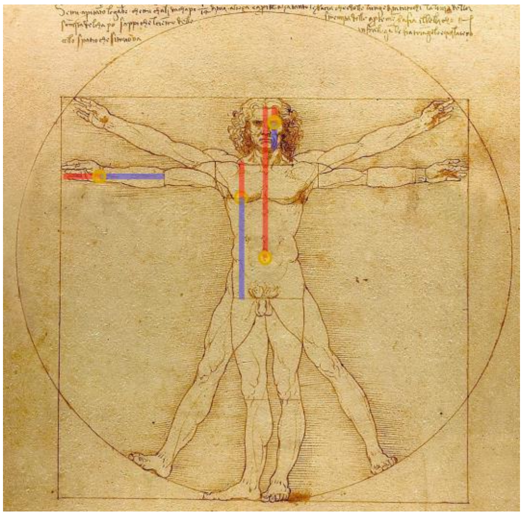
Certain facial features adhere to the golden ratio. For example, the ratio of the width of the mouth to the distance between the eyes is suggested to be close to the golden ratio, contributing to perceptions of facial harmony. Students might come across that facial mask that surgeons use in order to reconstruct a face. The facial mask is full with Golden Ratio analogies and students can really engage in this and find examples of faces that appear more beautiful after the mask is applied on them.

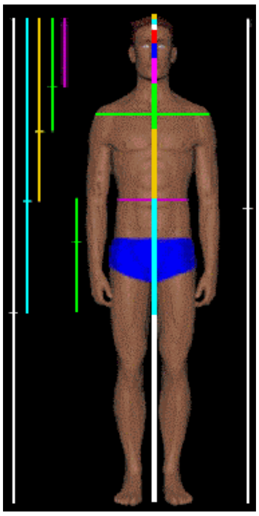




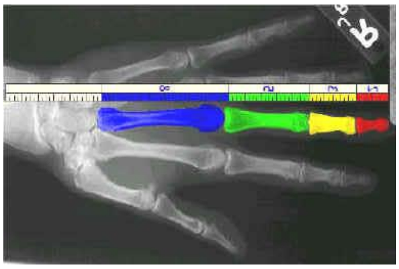
Original Revise to Marquardt Beauty Mask Per Beauty Mask

Body Proportions:
Present the notion that the golden ratio is proposed as a guideline for ideal body proportions. For instance, the ratio of the height to the navel compared to the navel to the feet is suggested to approximate the golden ratio. Explore how this concept may influence cultural ideals of beauty.





Hand and Fingers:
Explore the suggestion that the proportions of fingers and hand bones follow the golden ratio. For example, the length of each finger joint relative to the next is proposed to correspond to the golden ratio.



Skeletal Structure:
Introduce the idea that certain bones in the human body, such as those in the fingers and forearm, are proposed to follow the golden ratio. Discuss specific bone lengths and their supposed relationship to the golden ratio.

- *Conclusion:*

Conclude the lesson by summarizing key points and highlighting the complexity of beauty standards. Emphasize that the golden ratio is an interesting mathematical concept with a variety of applications.

“DO NOT GET DRUNK”

In the coastal town of Limassol, on the sunny southern coast of Cyprus, the most popular brand of commandaria – KEO St. John – is produced to a recipe that is now protected by a legally enforced appellation. One distinguishing feature of commandaria is that after the grapes are picked, they are left in the sun for ten days, which increases the density of their sugars. The grapes are then pressed, the wine is fortified (usually with a high percentage grape-based alcohol) and then it is aged for at least two years in oak barrels before being bottled. As the years roll by, the amber liquid intensifies in both viscosity and sweetness.

In this activity students will decide which is the most suitable areas in Cyprus for vine cultivation and they will try to make wine using grapes. The same lesson plan may be adjusted for any variety of grapes.

Expected outcome:

Students will make wine and conclude that the best area for growing vines in Cyprus is Limassol.

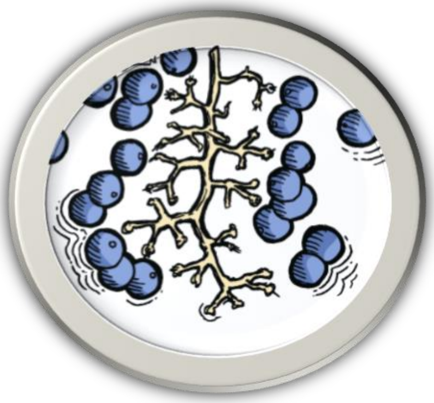
Questions to be investigated:

1. How wine is produced.
2. How Commandaria is produced.
3. Ideal location for vine growth.

Guidelines for Teachers:

1. **How wine is produced.**

Students will go through a webpage regarding the production of wine. The teacher will bring some red grapes to the class. Along with students they can try to make wine in the class. Firstly, they will wash the grapes and remove any leaves and branches from the grapes.



Then they crush the grapes with their hands, allowing the juice to escape. When they finish crushing the grapes, they will add some yeast that will start consuming the sugar from the mixture.

The last step is to place the mixture in a container and store it at a dark spot that is approximately 18^{o}C temperature.

The mixture needs to be regularly stirred to submerge the skins that color the wine red. Most wines take 5–21 days to ferment sugar into alcohol.
After that period, the mixture can be filtered and separate the wine from the skins.

More details can be found at
https://winefolly.com/deep-dive/how-is-red-wine-made/

2. How Commandaria is produced

Making commandaria is a bit different procedure than making wine. The indigenous grape varieties of Mavro (red) and Xynisteri (white) are picked late and dried in the sun, for ten days, to enhance their sugar content, giving the drink its distinctive taste.

The dried grapes are then pressed, with the run-off collected and fermented in tanks or huge earthenware jars - much like those used in bygone times and then it is aged for at least two years in oak barrels before being bottled.

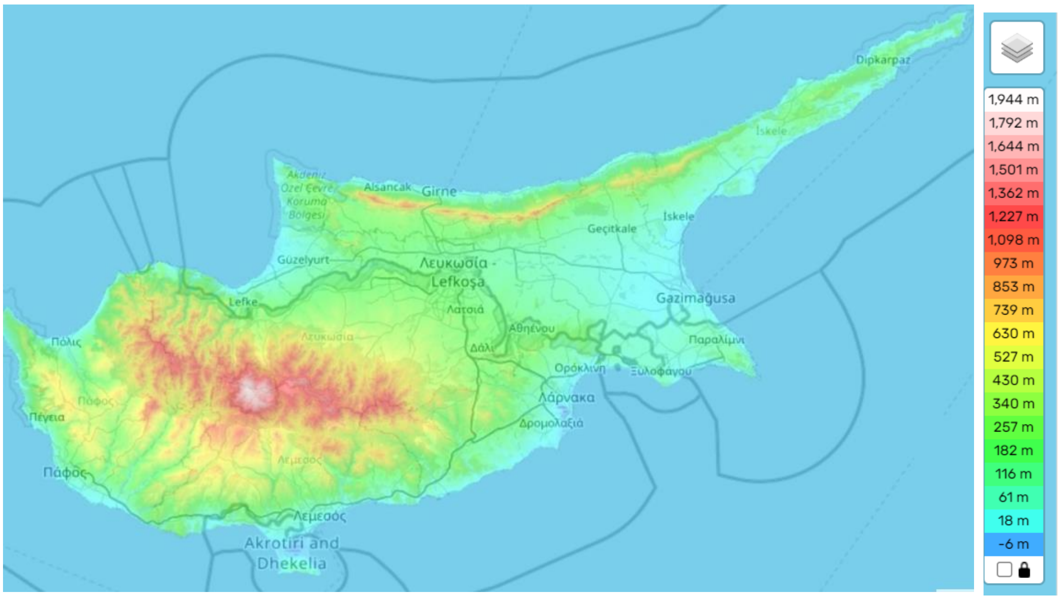
3. Ideal location for vine growth.

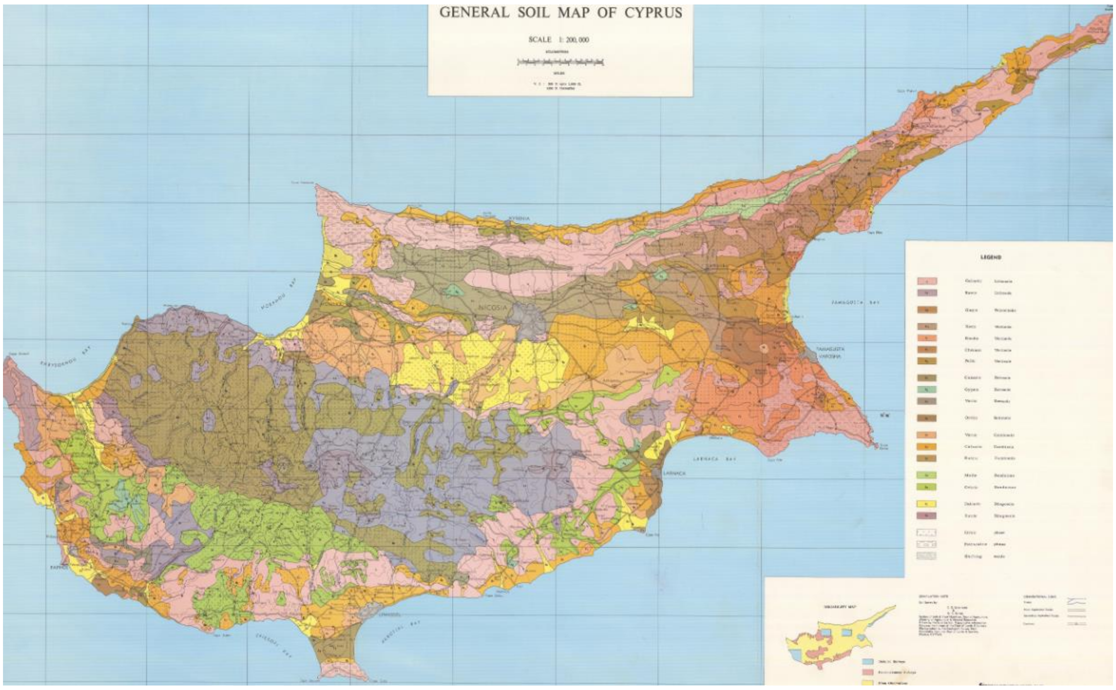
Students will study the following information regarding the ideal conditions for vine growth and after studying the maps they will propose possible locations for vine growth in Cyprus.

“For the cultivating vines, the direct Sun light exposure for more than six hours each day is a crucial factor. Also, organically rich soil is essential. If you find earthworms, the soil is in good shape.”

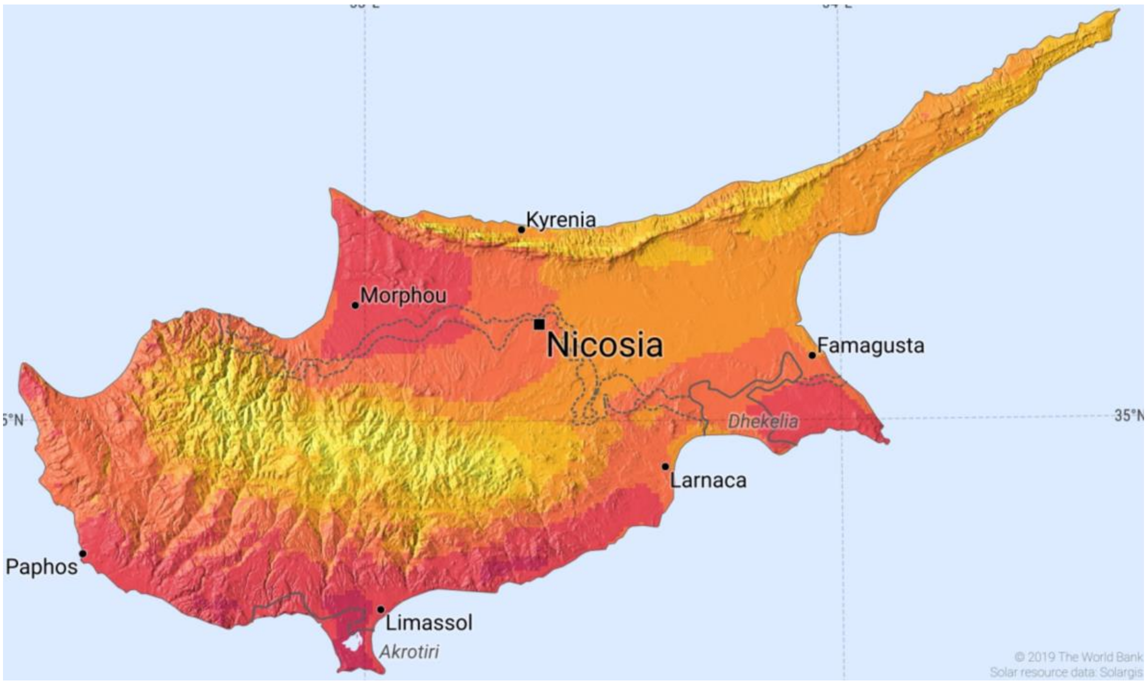
“These conditions of high temperature, sunshine and reduced humidity during summer, the altitude, the ground soil, and the unique method of processing the grapes, combine to give Commandaria its very special flavor and aroma. A height of approximately 750 meters above sea level is preferable.”

Then students are asked to use the following maps, and decide which areas are suitable for Commandaria growth in Cyprus.





Ideal soil for vine growth:



Sunlight per year: Lowest Highest

Conclusion

The ideal location for vines is the area of Limassol because it has the ideal amplitude, soil and sunlight exposure.